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A Sub-Millimeter Accurate Microwave Multilevel Gauging System for Liquids in Tanks

Matthias Weiß and Reinhard Knöchel

***Abstract*—A microwave multilevel gauging system employing a frequency-stepped continuous-wave radar measurement technique is described in this paper. A conventional frequency-modulated continuous-wave radar technique is normally employed only to find the level of the liquid surface in storage tanks. The system described here also detects a second level, e.g., the tank floor or an impurity level. If this second reflection dominates, distance measurement with the inverse Fourier transform (IFT) results in poor resolution and shows a very high range error for small gaps between these two scatterers. For estimating the exact time delay and amplitude of the reflection from each scatterer, an optimal signal-processing algorithm is derived, based on a reference model. Performance of the multiple target-detection reference model is illustrated using measured data obtained with an HP-8510 network analyzer. It is demonstrated that the reference model offers a significant enhancement of resolution over the standard processing IFT algorithm and is insensitive to noise and clutter signals approach. The described system achieves a time-delay accuracy with a bandwidth of $\Delta f = 1$ GHz, which corresponds to a range error of ± 0.2 mm.**

***Index Terms*—FSCW radar, gauging, level measurement, microwave measuring, multilevel measurement, permittivity.**

I. Introduction

Measurement and control of liquid levels in storage tanks and processing vessels is important in many industrial processes. In storage tanks containing liquids with low transmission losses, e.g., mineral oil, reflection coefficient measurement shows a strong signal from the bottom of the tank. This unwanted dominant reflection in the radar channel increases the detectable minimum liquid level and raises the range error of the determined liquid surface. In these circumstances, one uses a gauging system with a sensor terminated by a matched load [1].

All industrial level gauging systems are limited to a finite bandwidth due an inexpensive production, which leads to an attractively price. The resolution offered by the inverse Fourier transform (IFT) for a narrow bandwidth is often unsatisfactory [2] and an alternative approach is required. Even if there exists no such restricted limitation in bandwidth, maximization of the achievable resolution for a particular measurement bandwidth is important.

In this paper, a microwave frequency-stepped continuous-wave (FSCW) radar system is described, which can monitor the liquid surface and the bottom of the tank or an impurity level within the tank simultaneously. To extract the time delays τ_1 (liquid surface) and τ_2 (bottom) accurately from the measured data, a multitarget reference model algorithm is used. This evaluation procedure is derived in the following section.

The benefits of the reference model over normally used evaluation algorithms, e.g., the IFT, is that deviations from an ideal scatterer like dispersion and windowing can be taken into account [3]. The accuracy of the determined range can then be made as that of an ideal target. In contrast to the decreased range resolution of the IFT for a windowed spectrum, the multitarget reference model resolves the delays of two adjacent scatterers with the same precision.

The resolution limitations of the IFT approach are demonstrated along with the enhancement offered by the reference model using physical measurements made with an HP-8510 network analyzer.

II. Derivation of the Reference Model

An FSCW system transmits a sequence of sinusoids at different frequencies and measures the steady-state amplitude and phase shift induced by the radar channel [4]. Fig. 1 shows a block diagram of such a radar system. A significant benefit of performing the measurements at discrete frequencies is that digital signal processing may be easily applied to the data. To maximize the range resolution achievable from an FSCW radar, a reference model technique is used. By this technique, a signal-processing computer produces a set of synthesized data at frequencies where the measurements were taken, and compares it to the physical measurements. The computerized data are based on a physical model of the transfer function of the radar channel [5], [6].

Finally, the algorithm has to minimize the difference between both the measured and synthesized data. After minimization is performed, the parameters of the reference model represent the ranging results. Best results were achieved with the least squares estimate, described by

$$FF = \sum_{k=\mathrm{k}_0}^{\mathrm{k}_0+\mathrm{N}} [M_k - V_k] \cdot [M_k - V_k]^* \tag{1}$$

where the M_k are the measured complex reflection coefficient pairs, V_k are the values of the reference model, and k is the index number of each measurement, which spans the integer range from k_0 to $k_0 + \mathrm{N}$. FF is the error function, which depends on the parameters of the reference model as yet not specified, and * denotes the complex conjugate.

Manuscript received August 13, 1999.
The authors are with the Technische Fakultät der Christian-Albrechts-Universität zu Kiel, Lehrstuhl für Hochfrequenztechnik, D-24143 Kiel, Germany.
Publisher Item Identifier S 0018-9480(01)01081-X.

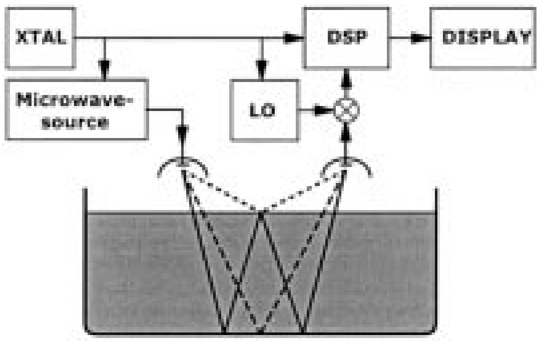

Fig. 1. Schematic diagram of an FSCW radar measuring a liquid gauge in a storage tank.

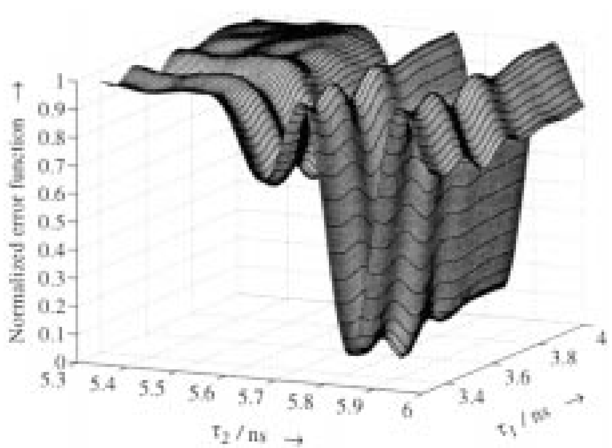

Fig. 2. Normalized error function constructed from a reflection-coefficient measurement in a tank with two independent delay times $\tau_1 =$ fluid level and $\tau_2 =$ tank bottom.

The physical description of the radar channel that will be assumed contains a number of reflective targets. The simplest realistic model for the system impulse response has the form

$$h(t) = \sum_i a_i \delta(t - \tau_i) \quad \circ\!\!-\!\!\bullet \quad H_k = \sum_i A_i e^{-j\Omega_k \tau_i} \tag{2}$$

where a_i is the reflection amplitude, A_i is the reflection coefficient, and τ_i are the time delays of the ith discontinuity. $\Omega_k = k \cdot \Delta\Omega$ ($k = k_0, \ldots, k_0 + \mathrm{N}$) denote the measured frequency. For liquids where $\epsilon_r' \gg \epsilon_r''$, the phase shift of the reflected impulse shows only a phase shift of 0° or 180° with respect to the incident signal, which means the amplitude of the echo is real. Therefore, the formulation of the reference model taking L reflections into account is

$$V_k(A_i, \tau_i) = \sum_{i=1}^{\mathrm{L}} A_i e^{-j\Omega_k \tau_i}. \tag{3}$$

The distance l_i of the reflection in the reference model leads to the time delay $\tau_i = 2l_i/c$.

Rewriting (3) in matrix form yields

$$\mathbf{V}(\mathbf{A_i}, \tau_\mathbf{i}) = \mathbf{Za} \tag{4}$$

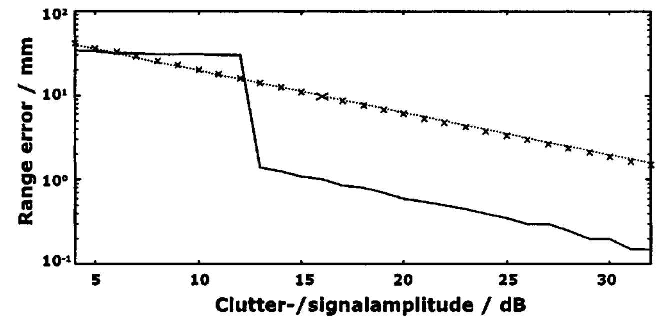

Fig. 3. Maximum range error caused by a clutter signal (×: theoretical behavior, - - -: IFT, —: reference model).

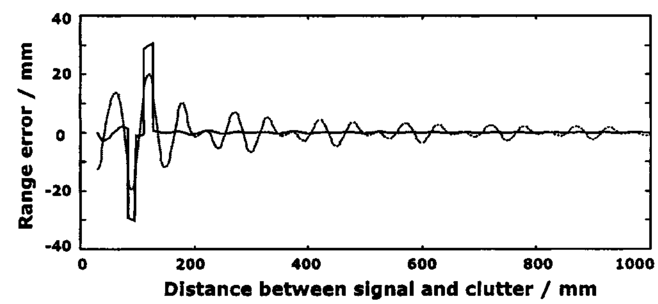

Fig. 4. Comparison of two algorithms for the distance measurement with a clutter amplitude $a = 10$ dB below the target signal (—: reference model and - - -: IFT).

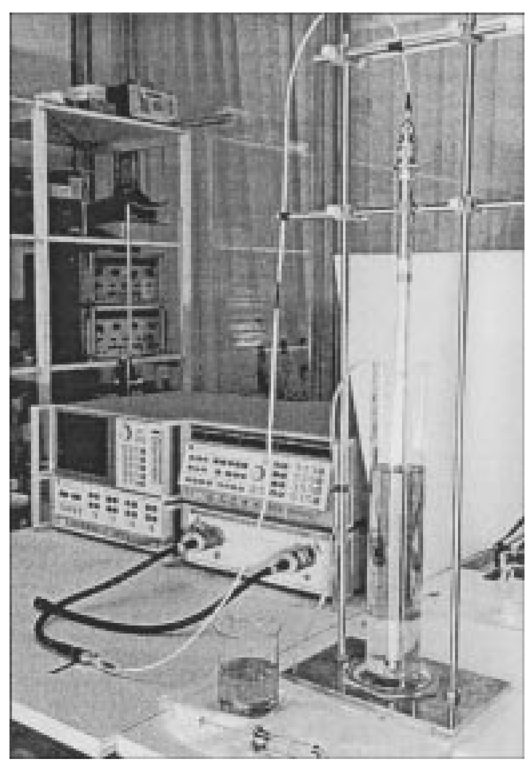

Fig. 5. Experimental setup (probe terminated with a short). The fluids are diesel and water, which are clearly separated.

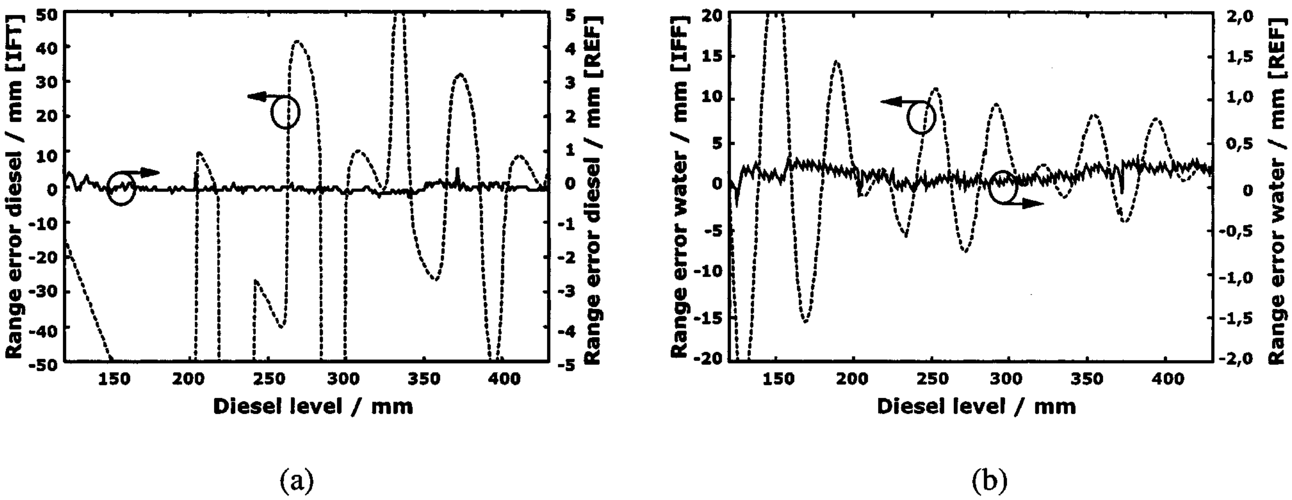

Fig. 6. Comparison of two algorithms for distance measurement. The reflection signal from the bottom is about 14 dB higher than that from the liquid surface. The dotted line shows the result of the IFT and the solid line of the reference model (Δf $=$ 1 GHz). (a) Achieved accuracy for the diesel level. (b) Achieved accuracy for the water level.

with $\mathbf{a} = [\mathbf{A_1}\, \mathbf{A_2}, \ldots, \mathbf{A}_L]^\mathbf{T}$ the amplitude vector and the $(N \times L)$ matrix $\mathbf{Z}$, which describes the frequency response of each target as follows:

$$\mathbf{Z} = \begin{bmatrix} \exp(-j\Omega_1\tau_1) & \exp(-j\Omega_1\tau_2) & \cdots & \exp(-j\Omega_1\tau_L) \\ \vdots & \vdots & \ddots & \vdots \\ \exp(-j\Omega_N\tau_1) & \exp(-j\Omega_N\tau_2) & \cdots & \exp(-j\Omega_N\tau_L) \end{bmatrix}.$$

The measured reflection coefficients are then represented by the following vector:

$$\mathbf{M} = \Big[M(\Omega_1)\; M(\Omega_2), \ldots, M(\Omega_N)\Big]^T. \tag{5}$$

With these abbreviations, the general least squares optimization problem yields in the matrix description

$$\begin{aligned} FF(A_i, \tau_i) &= |\mathbf{M} - \mathbf{V}|^\mathbf{2} \\ &= |\mathbf{M} - \mathbf{Za}|^\mathbf{2} \\ &= \Big|(\mathbf{M_r} + \mathbf{jM_i}) - (\mathbf{Z_r} + \mathbf{jZ_i})\mathbf{a}\Big|^\mathbf{2} \end{aligned} \tag{6}$$

where $|\cdot|$ denotes the sum of the vector. Assuming real reflection amplitudes in (6) results in the following squared inner product:

$$\begin{aligned} FF(A_i, \tau_i) = &(\mathbf{M_r} - \mathbf{Z_r a})^T(\mathbf{M_r} - \mathbf{Z_r a}) \\ &+ (\mathbf{M_i} - \mathbf{Z_i a})^T(\mathbf{M_i} - \mathbf{Z_i a}). \end{aligned}$$

Using the following abbreviations:

$$\begin{aligned} E_M &= \mathbf{M_r^T M_r} + \mathbf{M_i^T M_i} \\ \mathbf{H} &= \mathbf{Z_r^T Z_r} + \mathbf{Z_i^T Z_i} \\ \mathbf{b} &= \mathbf{Z_r^T M_r} + \mathbf{Z_i^T M_i} \end{aligned}$$

the error function is

$$FF(A_i, \tau_i) = E_M + \mathbf{a^T H a} - \mathbf{a^T b} - \mathbf{b^T a}. \tag{7}$$

To minimize the error function $FF(A_i, \tau_i)$ analytically the derivative of (7) is taken with respect to the amplitude vector $\mathbf{a}$ and set equal to zero as follows:

$$\frac{\partial FF}{\partial \mathbf{a}} = 0 = \mathbf{Ha} + \mathbf{a^T H} - \mathbf{b} - \mathbf{b^T}. \tag{8}$$

From this solution, it is possible to determine the amplitude vector $\mathbf{a}$. The transpose of the symmetric matrix $\mathbf{H}$ is the same as the original, i.e., $\mathbf{H^T} = \mathbf{H}$. Therefore, the determination of the amplitude vector yields

$$\mathbf{a} = \mathbf{H^{-1} b}. \tag{9}$$

Inserting (9) in (7) gives

$$FF(\tau_i) = E_M - \mathbf{b^T H^{-1} b}. \tag{10}$$

When the reference model matches the measurement, the error function $FF(\tau_i)$ has an absolute minimum.

In certain cases, however, the determination of the amplitude vector $\mathbf{a}$ from the inverse of the $\mathbf{H}$ matrix is extremely ill conditioned. This happens when the time delays of two reflectors are nearly equal ($\tau_1 \approx \tau_2$). When the values of τ_1 and τ_2 are exactly equal, the symmetric matrix $\mathbf{H}$ becomes singular. The following example for two reflection targets shows this behavior:

$$\begin{aligned} \mathbf{H_{11}} = \mathbf{H_{22}} &= \sum_k \cos^2(\Omega_k\tau_1) + \sin^2(\Omega_k\tau_1) = \mathrm{N} \\ \mathbf{H_{12}} = \mathbf{H_{21}} &= \sum_k \cos(\Omega_k\tau_1)\cos(\Omega_k\tau_2) \\ &\quad + \sin(\Omega_k\tau_1)\sin(\Omega_k\tau_2). \end{aligned}$$

To solve the multidimensional optimization problem of (10) in a reasonable time, the following procedure has proven to be efficient:

1) estimate a number of existing reflection targets L;
2) assume a sequence of delays τ_i and their possible range and dependence (multiple reflections);
3) search for the global minimum of the error function expressed by (10) dependent on the delays τ_i.

It can be shown for a two-dimensional problem that the error function in (10) can be interpreted as a surface stretched over the variables. The multiple minima surface shows grooves aligned with the delay axes. The grooves in this surface can be searched sequentially for each reflector. Afterwards, the global minimum, including all existing reflectors, is found with little computing. The global minimum corresponds to the set of optimum time delays and, hence, to the target ranges.

Fig. 2 shows such a surface of the error function. The analyzed measurement was carried out in a storage tank. The distance between the antenna and diesel level corresponds to a time delay of $\tau_1 = 3.5$ ns. The second reflector, the dominant echo signal from the tank bottom, corresponds to a time delay of $\tau_2 = 5.8$ ns. Measurements of the reflection coefficient were taken over the frequency range from 1 to 4 GHz using 801 evenly spaced frequencies.

III. Range Error Caused by a Clutter

In this section, we determine the maximum range error caused by clutter not considered in the reference model, e.g., a ladder in a tank. For most of the evaluation algorithms, this range error can be estimated because it only depends on the ratio of the amplitudes of clutter to target signal and on the frequency bandwidth. This estimation does not give all information about the quality incidence of the error or about the robustness of the algorithm. It represents the *worst case* only.

If the reflection-coefficient measurement is evaluated by the IFT, the derivation of the maximum range error takes place in the time domain. The determined range error caused by clutter having the normalized amplitude a is expressed by the following relationship:

$$\Delta l \leq 0.209\, \frac{c}{\Delta \mathrm{f}}\, a. \tag{11}$$

To determine the maximum range error occurring by an unknown clutter on the radar channel a simulation was carried out. At a bandwidth of 1 GHz, the amplitude ratio a, clutter to target, rises from -4 to -32 dB in 1-dB steps. Fig. 3 shows the resulting error for one reflector in the reference model (dashed line). The crosses represent the theoretical behavior of the IFT described by (11) and the continuous line shows the IFT results from the simulation.

It should be emphasized that the determined maximum range error when applying the reference model is about a factor of ten better then that of the IFT for low clutter signals ($a < -12$ dB). This behavior changes when the ratio rises above $a > -12$ dB. The distance error from the reference model then increases and remains high. The reason for that behavior is that the IFT transforms a rectangle in the frequency range to an si impulse in the time domain. The same signal evaluated by the reference model yields an error function $FF(\tau)$, which has a couple of deep and sharp minima. The amplitude of these minima follows an inverted si function. A nearby clutter influences this *fence* by slightly shifting the minima and varying the amplitude. At a specific clutter-to-signal ratio and distance, a nearby minimum becomes the absolute minimum.

Simulations have shown that this maximum range error for the reference model only occurs at two distance ranges between the target and clutter. Fig. 4 shows the calculated range error for the reference model (continuous line) and the IFT (broken line) over the distance of two reflectors. The simulation bandwidth was 1 GHz and the amplitude of the *unknown* reflector was $a = 10$ dB below the target signal. The diminished oscillating range error for the IFT caused by the sideband of the si function in the time domain is clearly recognizable. If the distance measurement is carried out using the reference model (solid line), the error is much lower, except for two distinct ranges.

IV. Experimental Results

Fig. 5 shows the experimental setup. The coaxial sensor, which has a length of 850 mm, stands in a glass cylinder filled with diesel ($\epsilon_r = 2.15$) and water. Both fluids are clearly separated, as it would be in a storage tank after a long time period. The sensor is terminated by an absorber to eliminate unwanted reflections from the tank bottom. The FSCW measurements were carried out using an HP8510C network analyzer in the frequency range from 2 to 3 GHz. The diesel level was changed in 1-mm steps. In this way, measurements were taken in the range from 50 to 355 mm above the surface of the water.

In Fig. 6, a comparison of two range algorithms is shown. The dotted line was processed by the conventional technique, i.e., performing the IFT and determining the time delays. The solid line is the result of the comparison between the measured data and the reference model. Its variation lies within the limits of approximately ±0.2 mm.

For two identical strong echoes, the range resolution of the IFT can be determined by [3]

$$\Delta l = c\Delta \mathrm{f}. \tag{12}$$

For a frequency bandwidth $\Delta \mathrm{f} = 1$ GHz, the estimated resolution of the IFT is $\Delta l = 300$ mm. If the power of the target reflection is less than that of the clutter, the range resolution offered by the IFT decreases.

V. Conclusions

A microwave-level gauging system suitable for industrial applications requiring sub-millimeter accuracy has been described in this paper. Its accuracy is achieved even if a strong reflection signal from the tank bottom exists.

The algorithm (reference model) used for determining the time delays of the different scatterers is based on minimization of the difference between the measured reflection data and those produced by a reference model by least squares estimation. This model assumes that the echoes from the radar channel are pulses in the time domain. The algorithm does not require a constant interval between the measurement samples.

Sensitivity studies were presented with respect to the influence of a clutter signal. A performance comparison between IFT and the reference model has been presented. Measurements were carried out employing a coaxial sensor terminated by a short using an HP8510C network analyzer. The IFT was shown to produce useless level gauging signals for the surface if a strong reflection from the end of the sensor existed. This can be a water layer or a bottom plate in petrol tanks. Using the reference model, it was possible to resolve the time delays of the first two reflections with high accuracy.

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